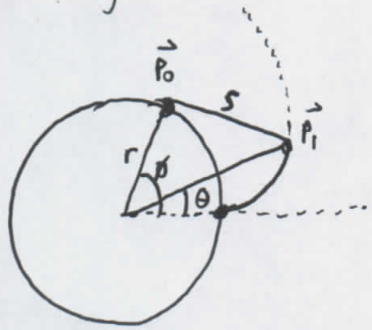
Cleaned Up Notes: Involute Gears

12 July 2022

Geometry of an Involute curve



$$s = r\varnothing$$
$$\vec{p_0} = r[\cos(\varnothing), \sin(\varnothing)]$$
$$\vec{p_1} = p_0 + s[\sin(\varnothing), -\cos(\varnothing)]$$

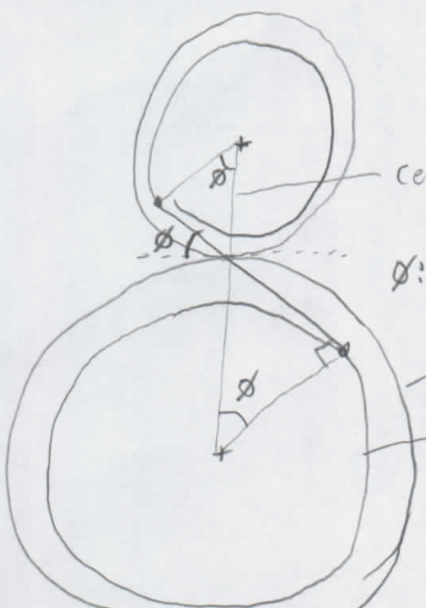
$$x^* = \frac{x}{r} = \cos(\varnothing) + \varnothing\sin(\varnothing)$$
$$y^* = \frac{y}{r} = \sin(\varnothing) - \varnothing\cos(\varnothing)$$

$$r_i = |[x,y]|$$
$$\theta_i = \mathrm{atan2}(y,x)$$

$$x^* = \cos(\varnothing-\theta_0) + \varnothing\sin(\varnothing-\theta_0)$$
$$y^* = \sin(\varnothing-\theta_0) - \varnothing\cos(\varnothing-\theta_0)$$

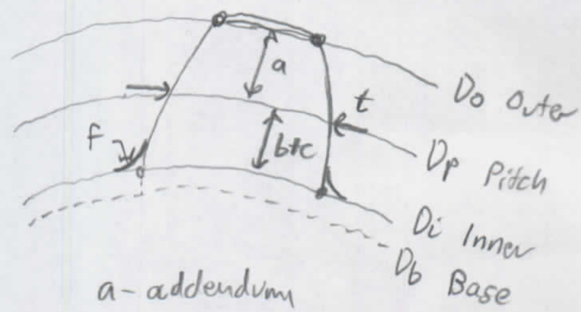
$$dx^*/d\varnothing = \cos(\varnothing-\theta_0)\cdot\varnothing$$
$$dy^*/d\varnothing = +\sin(\varnothing-\theta_0)\cdot\varnothing$$

Gear Geometry



Ø: Pressure Angle

$$D_b = D_p\cos(\varnothing)$$



a - addendum
b - dedendum
f - root fillet diameter
c - clearance

P: Diametral Pitch

$$\frac{\#\text{ of teeth}}{\text{base length}} \qquad \frac{\#\text{ of teeth}}{1\text{ inch gear}} \qquad D_p = \frac{N}{P}$$

48:24 Gear: $\frac{48\text{ teeth}}{1\text{ inch diameter}}$: ($\frac{1}{2}$" diameter, 24 teeth)

More Gear Geometry

Common Pressure Angles

$\phi = 20^\circ$ New gears

$\phi = 14.5^\circ$ Older gear standard

ANSI B6.1-1968 (R1974) American National Standard Gear Tooth Forms

ASA B6.1-1932 Former American Standard Gear Tooth Forms

Ref Machinery's Handbook 28 pp 2043+

$a = \frac{1.000}{P}$ Addendum

$b = \frac{1.200}{P} + 0.002$ Dedendum

(a, b) match all gears in a particular diametral pitch series

$D_p = \frac{N}{P}$

$C_p = \frac{\pi D_p}{N} = \frac{\pi}{P}$ Circular Pitch

$c = \frac{0.200}{P} + 0.002$ Clearance

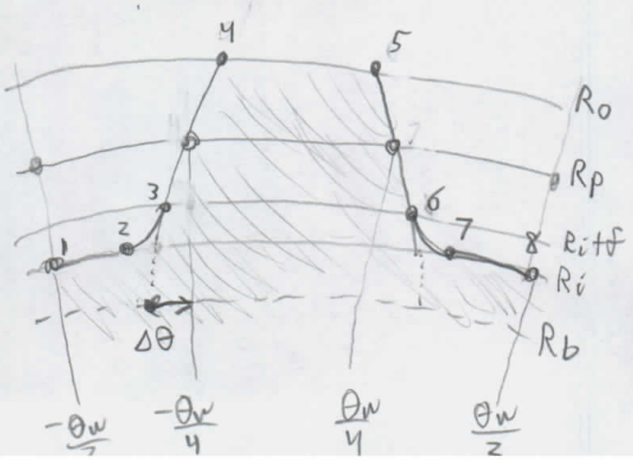
$f = \frac{0.300}{P}$ Fillet Radius

$D_b = D_p \cdot \cos(\phi)$ Base Diameter

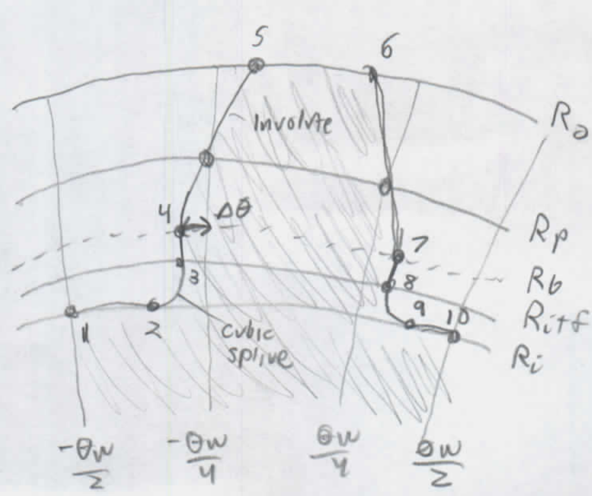
$D_o = D_p + 2a$ Outer Diameter

$D_i = D_p - 2b - 2c$ Inner Diameter

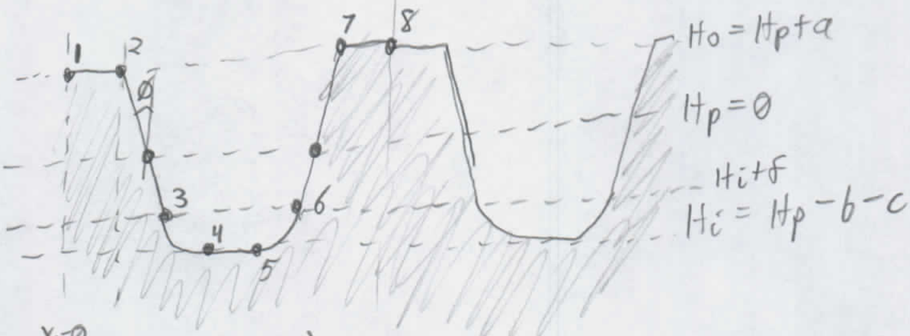
Case: $R_b < R_i + f$



Case $R_b \geq R_i + f$



Rack Gear Geometry



$x_1 = \emptyset$

$x_2 = \frac{c_p}{2} - a \cdot \sin(\emptyset)$

$x_3 = \frac{c_p}{2} + (b - f)\sin(\emptyset)$

$x_4 = \frac{c_p}{2} + b\sin(\emptyset) + f$

$x_5 = c_p - x_4$

$x_6 = c_p - x_3$

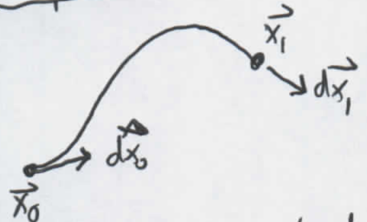
...

$c_p = \frac{\pi}{P}$

a, b, c, f calculated as before

Cubic Spline

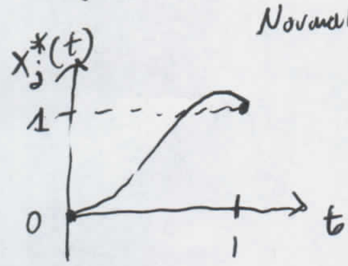
(Normalized so that scaling $\vec{x}$ does not change relative shape)



$(\vec{x}_1, d\vec{x}_1, \vec{x}_2, d\vec{x}_2, \underbrace{r_1, r_2}_{\text{bendyness of curve}} \approx 1.5 \text{ works})$

$d\vec{x}_i^* = \frac{d\vec{x}_i}{|dx_i|} \cdot r_i$ normalized rate

Normalized coordinates



$L_j = (\vec{x}_2 - \vec{x}_1)_j$ Lengthscale for each dimension

$(x^*_i)_j = \frac{(x_i)_j - (x_1)_j}{L_j}$ normalized coordinates

$(d\vec{x}_i^*)_j = -(d\vec{x}_i^*)_j$ if $L_j < \emptyset$ normalized rate of change

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ dx_1^* \\ dx_2^* \end{bmatrix}$$

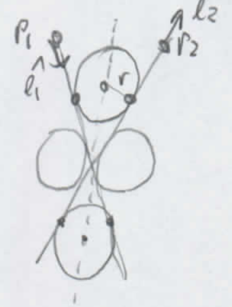
for each dimension j

$$x^*(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$x(t)_j = x^*(t)_j \cdot L_j + x_{1j}$$

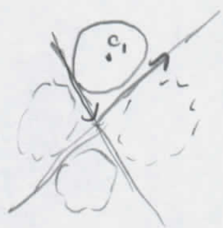
Fillet Problem: Given two lines and a radius find the circle

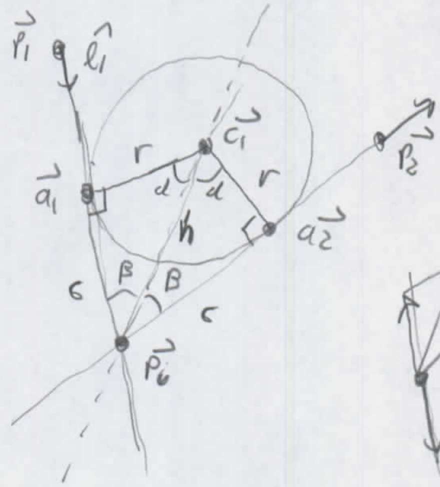
$$(\vec{p_1}, \hat{l_1}, \vec{p_2}, \hat{l_2}, r) \to (\vec{c}); \vec{a_1}, \vec{a_2}$$

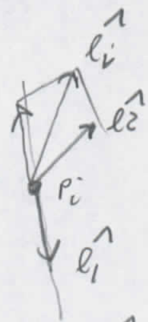


Can avoid solving a nonlinear problem by looking closer at the geometry of the ~~of~~ kite shape formed by a valid circle

Oriented solution picks $\vec{c_1}$ out of $\{\vec{c_i}\}$



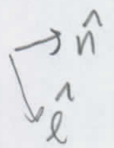




Line of intersection bisects figure

$\hat{n} = \hat{l}^*$ hodge dual

$(n_x, n_y) = (-l_y, l_x)$



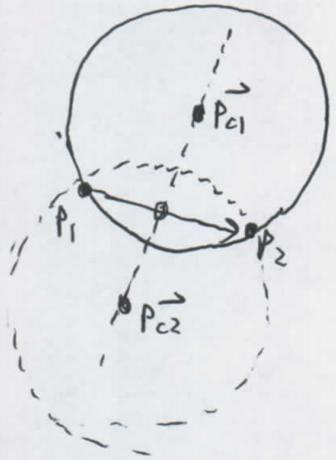
$$\hat{l_i} = \frac{-\hat{l_1} + \hat{l_2}}{|\vec{l_i}|}$$

$$\vec{c_1} = \vec{p_i} + \hat{l_i} h = \vec{c_1} + l_i \cdot \frac{r}{\sin(\beta)}$$

$$\beta = \frac{1}{2} \operatorname{asin}(\hat{l_2} \times (-\hat{l_1}))$$
$$= \frac{1}{2} \operatorname{asin}(\hat{l_1} \times \hat{l_2})$$

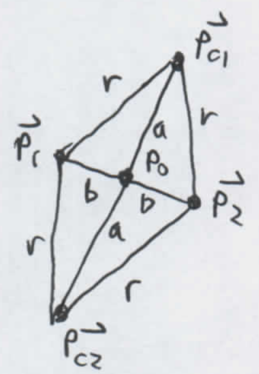
$$l_1 \times l_2 = l_{1x} l_{2y} - l_{1y} l_{2x}$$

Circle from two points and a radius



Two solutions - If I want, I can choose one of them in an oriented fashion from the sense of $\vec{l} = \vec{P_2} - \vec{P_1}$

Given $(\vec{P_1}, \vec{P_2}, r)$ Find $(\vec{P_c})$



$$\vec{P_0} = \frac{1}{2}(\vec{P_1} + \vec{P_2})$$

$$\hat{l} = \frac{\vec{P_2} - \vec{P_1}}{|P_2 - P_1|}$$

$$\hat{n} = \hat{l}^* \qquad (n_x, n_y) = (-l_y, l_x) \text{ Hodge Dual}$$

$$b = |P_2 - P_1|/2$$

if $b < r$

$$\begin{cases} a = \sqrt{r^2 - b^2} \\ \vec{P_{c1}} = \vec{P_0} + a \cdot \hat{n} \\ \vec{P_{c2}} = P_0 - a \cdot \hat{n} \end{cases}$$